Scaling Limit of Longitudinal Virtual Compton Cross Sections

Jeffrey E. Mandula*
California Institute of Technology, Pasadena, California 91109
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It is shown that the hypothesis that the quark model gives the correct leading, $\delta'(x^2)$, light-cone singularity of current commutators requires that R, the ratio of longitudinal to transverse virtual Compton cross sections, go to zero like $1/\nu$ in the Bjorken limit.

The light-cone current algebra abstracted from the quark model[1] has been applied to deep-inelastic electron scattering to explain the scaling observed in the SLAC-MIT experiment.[2] The algebra implies scaling for the structure functions[3] mW_1 and νW_2 in the deep-inelastic region, that is, it asserts that for large ν and $-q^2$ these structure functions should become functions only of the dimensionless variable $\xi=-q^2/2m\nu$. The algebra also implies that the linear combination

$$W_L \equiv \left(1-\frac{\nu^2}{q^2}\right)W_2 - W_1 \tag{1}$$

should vanish in the deep-inelastic region.[4] The purpose of this note is to point out that the quark light-cone algebra further determines that the rate at which W_L goes to zero is like $1/\nu$. Put differently, it implies scaling for νW_L:

$$\lim_{\substack{-q^2,\nu\to\infty \\ \xi=-q^2/2m\nu \text{ fixed}}} \nu W_L(\nu,q^2)\to h(\xi)\,. \tag{2}$$

This scaling prediction is quite independent of the scaling of mW_1 or νW_2 and provides a new test of the algebra. Its experimental verification requires more precise measurements of $R=W_L/W_1$, the ratio of the longitudinal to the transverse virtual Compton scattering cross sections.

The scaling of νW_L follows from the quark light-cone algebra (and so from the parton model[5] in which the algebra holds), but, in contrast with other scaling predictions, it cannot be deduced from dimensional considerations alone.[6-10] The statement that the quark model gives the operator structure of the leading light-cone singularity has implications for nonleading behavior in the deep-inelastic region, that is, for dimensionally nonleading singularities. The vanishing of W_L like $1/\nu$ in the scaling region has been conjectured before by Yao and Mahanthappa[11] and shown to hold canonically in the quark-gluon model by Dicus, Jackiw, and Teplitz,[12] but it has not been recognized as a necessary conclusion from hypothesis that the quark model gives the leading light-cone singularity. Positivity restrictions on the structure functions for polarized as well as unpolarized inelastic electron scattering[13] together with the naive dimensional scaling properties for these functions suggest that W_L should fall no faster than $1/\nu$ in the scaling region.

To derive the result one need only notice that

$$\frac{1}{m}p_\mu p_\nu W_{\mu\nu}=m\left(1-\frac{\nu^2}{q^2}\right)W_L\,, \tag{3}$$

which becomes $(\nu/2\xi)W_L$ in the deep-inelastic region. So evaluation of $(1/m)p_\mu p_\nu W_{\mu\nu}$ from the leading light-cone singularity of $[J_\mu(x),J_\nu(0)]$ gives νW_L in the scaling region directly.

In the quark model one has for that singularity

$$[J_\mu(x),J_\nu(0)]_{(\text{symmetric in }\mu\nu)} = \frac{1}{2\pi}\delta'(x^2)s_{\mu\nu\lambda\sigma}\epsilon(x_0)x_\lambda[\mathfrak{F}_\sigma(x,0)-\mathfrak{F}_\sigma(0,x)]+\cdots\,, \tag{4}$$

where the bilocal density is

$$\mathfrak{F}_\sigma(x,0)=\tfrac{2}{3}\mathfrak{F}^3_\sigma(x,0)+\frac{2}{3\sqrt{3}}\mathfrak{F}^8_\sigma(x,0)+\frac{4\sqrt{2}}{3\sqrt{3}}\mathfrak{F}^0_\sigma(x,0)\,, \tag{5}$$

$s_{\mu\nu\lambda\sigma}=\frac{1}{4}\mathrm{Tr}\gamma_\mu\gamma_\lambda\gamma_\nu\gamma_\sigma$, and $\cdots$ denotes terms less singular than $\delta'(x^2)$ at $x^2=0$. Since each component has a $\delta'(x^2)$ singularity, each component of $W_{\mu\nu}$ will be finite in the Bjorken limit (if evaluated in a frame in which p remains finite). Thus one sees that νW_L scales in that limit.

To display the limiting function in terms of the light-cone singularity one defines generalized form factors of $\mathfrak{F}_\sigma(x,0)$ between spin-averaged states

$$\sum_{\text{spins}}\langle p|\mathfrak{F}_\sigma(x,0)|p\rangle=p_\sigma F(p\cdot x)+x_\sigma G(p\cdot x)+\cdots\,, \tag{6}$$

where $\cdots$ denotes terms which vanish at $x^2=0$.

These functions have the Fourier representations

$$\begin{aligned} F(p\cdot x)-F(-p\cdot x)&=\frac{4}{m}\int_{-1}^{1}e^{i\alpha p\cdot x}f(\alpha)d\alpha\,,\\ G(p\cdot x)+G(-p\cdot x)&=2im\int_{-1}^{1}e^{i\alpha p\cdot x}g(\alpha)d\alpha\,, \end{aligned} \tag{7}$$

where the limits are fixed by the spectral restrictions on $W_{\mu\nu}$. In terms of these one has

$$\lim_{\substack{-q^2,\nu\to\infty \\ \xi=-q^2/2m\nu \text{ fixed}}} \frac{1}{m} p_\mu p_\nu W_{\mu\nu} \to f(\xi) - g'(\xi) . \tag{8}$$

Comparison with Eq. (2) gives

$$h(\xi) = 2\xi(f(\xi) - g'(\xi)) . \tag{9}$$

The scaling function $f(\xi)$ is already known in deep-inelastic electron scattering. If the quark-model commutators are right, it is the Bjorken limit of W_1:

$$\lim_{\substack{-q^2,\nu\to\infty \\ \xi=-q^2/2m\nu \text{ fixed}}} mW_1(\nu, q^2) \to f(\xi) . \tag{10}$$

The other scaling function, $g(\xi)$, has not previously appeared, however. It comes from operators whose spin is two units below that of the leading spin operator in each coefficient in the Taylor-series expansion of $\mathfrak{F}_\sigma(x,0)$ in powers of x_μ. These higher twist operators give contributions to the matrix elements $\langle p|\mathfrak{F}_\sigma(x,0)|p\rangle$ proportional to x_σ and do not contribute to the Bjorken limits of either mW_1 or νW_2. For that reason they are ordinarily discarded,[1] but they can make a finite contribution to νW_L in the scaling limit. Were they absent one would have $(\nu/m)R = \nu W_L/mW_1 \to 2\xi$.

The fact that the quark light-cone algebra has definite implications for dimensionally nonleading singularities can be seen in the light-cone analysis of the current commutator[14]

$$[J_\mu(x), J_\nu(0)]_{(\text{symmetric in } \mu\nu)} = -(\Box^2 g_{\mu\nu} - \partial_\mu\partial_\nu)[E(x; C_L)\mathcal{O}(x,0) + \cdots] \\ + (\Box^2 g_{\mu\lambda} g_{\nu\sigma} - \partial_\mu\partial_\lambda g_{\nu\sigma} - \partial_\nu\partial_\sigma g_{\mu\lambda} + \partial_\lambda\partial_\sigma g_{\mu\nu})[E(x; C_T)\mathcal{O}^{\lambda\sigma}(x,0) + \cdots] . \tag{11}$$

Dimensional considerations require only that $C_T \leq 0$, $C_L \leq 2$, but for the quark algebra to give the correct leading, $\delta'(x^2)$, singularity for all components of the currents one needs $C_T = 0$, $C_L \leq 0$.

It is clear that the assumption that the quark model gives the leading light-cone singularity is stronger than the assumption that the scale-invariant part of $[j_\mu(x), j_\nu(0)]$ is given by the quark model. It is possible for a longitudinal light-cone singularity to be weaker than that allowed by scale invariance and yet be stronger than, or as strong as, $\delta'(x^2)$ at $x^2 = 0$. If longitudinal singularities stronger than $\delta'(x^2)$ are present then $(\nu/m)R$ will not scale, but another expression, $(\nu/m)^a R$, with $0 < a < 1$, will. If longitudinal singularities proportional to $(x_\mu x_\nu - g_{\mu\nu}x^2)\delta'(x^2)$, but no stronger, exist, then the scaling law for $(\nu/m)R$ would be preserved, but these singularities would make a finite correction to the scaling function.

Longitudinal singularities proportional to $(x^\mu x^\nu - g^{\mu\nu}x^2)\delta'(x^2)$ would be as strong on the light cone as the leading quark-model singularities. Their coefficients would be new bilocal operators beyond those familiar from the quark model.

Note that the scaling of νW_L is a property of the leading quark-model singularity and is independent of the nature of lesser singularities.

It should be remarked that these considerations also apply to neutrino scattering, and imply scaling for $\nu[-W_1 + (1 - \nu^2/q^2)W_2]$ in deep-inelastic ν and $\bar{\nu}$ scattering from any hadronic target. One obtains these same results for neutrino scattering because in the quark algebra the SU(3) structure factors [only the numerical coefficients of $\mathfrak{F}^i_\sigma(x,0)$ in Eq. (5) are changed] and the leading light-cone singularities of $[V^i_\mu(x), V^j_\nu(0)]$ and $[A^i_\mu(x), A^j_\nu(0)]$ are the same, while $[V^i_\mu(x), A^j_\nu(0)]$ contributes neither to W_1 nor W_2, but to W_3.

The present experimental knowledge of R is too imprecise to either verify or deny the scaling hypothesis for $(\nu/m)R$. Better measurements can be expected with the continuing refinement of the deep-inelastic electron scattering experiments at SLAC.

The vanishing of R like $1/\nu$ in the scaling region was first obtained by R. P. Feynman from the parton model. The author would like to thank Professor Feynman, Dr. R. Heimann, and Dr. A. Hey for several illuminating conversations.

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[3]The notation is p = 4-momentum of hadronic target, m = mass of hadronic target, q = 4-momentum trans-

ferred by current, $\nu = p\cdot q/m$, $\xi = -q^2/2M\nu$. The structure functions are defined by

$$W_{\mu\nu}(p,q) = \frac{1}{2\pi}\sum_{\text{spins}}\int d^4x\, e^{iq\cdot x}\,\langle p|\,[J_\mu(x), J_\nu(0)]\,|p\rangle$$
$$= -(g_{\mu\nu} - q_\mu q_\nu/q^2)\,W_1(\nu,q^2)$$
$$+\frac{1}{m^2}\,(p_\mu - q_\mu p\cdot q/q^2)$$
$$\times\,(p_\nu - q_\nu p\cdot q/q^2)\,W_2(\nu,q^2).$$

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